

Trigonometric Functions

Question 1.

The value of $\sin 15 + \cos 15$ is

- (a) 1
- (b) $1/2$
- (c) $\sqrt{3}/2$
- (d) $\sqrt{3}$

Answer: (c) $\sqrt{3}/2$

$$\begin{aligned}\text{Given, } \sin 15 + \cos 15 &= \sin 15 + \cos(90 - 15) \\ &= \sin 15 + \sin 15 \\ &= 2 \times \sin 45 \times \cos 30 \\ &= 2 \times (1/\sqrt{2}) \times (\sqrt{3}/2) \\ &= \sqrt{3}/2\end{aligned}$$

Question 2.

The value of $\tan A/2 - \cot A/2 + 2\cot A$ is

- (a) 0
- (b) 1
- (c) -1
- (d) None of these

Answer: (a) 0

$$\begin{aligned}\text{Given, } \tan A/2 - \cot A/2 + 2\cot A &= \{\sin(A/2)/\cos(A/2)\} - \{\cos(A/2)/\sin(A/2)\} + 2\cot A \\ &= \{\sin^2(A/2) - \cos^2(A/2)\} / \{\cos(A/2) \times \sin(A/2)\} + 2\cot A \\ &= -\{\cos^2(A/2) - \sin^2(A/2)\} / \{\cos(A/2) \times \sin(A/2)\} + 2\cot A \\ &= -\{\cos A\} / \{\cos(A/2) \times \sin(A/2)\} + 2\cot A \text{ (since } \cos^2 A - \sin^2 A = \cos 2A \text{)} \\ &= -\{\cos(2A/2)\} / \{\cos(A/2) \times \sin(A/2)\} + 2\cot A \\ &= -\{2 \times \cos A\} / \{2 \times \cos(A/2) \times \sin(A/2)\} + 2\cot A \\ &= -\{2\cos A\} / \{\sin(2A/2)\} + 2\cot A \\ &= \{-(2\cos A)/(\sin A)\} + 2\cot A \text{ (since } \sin 2A = 2 \times \sin A \times \cos A \text{)}\end{aligned}$$

$$= -2\cot A + 2\cot A$$

$$= 0$$

Question 3.

The value of $4 \times \sin x \times \sin(x + \pi/3) \times \sin(x + 2\pi/3)$ is

- (a) $\sin x$
- (b) $\sin 2x$
- (c) $\sin 3x$
- (d) $\sin 4x$

Answer: (c) $\sin 3x$

Given, $4 \times \sin x \times \sin(x + \pi/3) \times \sin(x + 2\pi/3)$

$$= 4 \times \sin x \times \{\sin x \times \cos \pi/3 + \cos x \times \sin \pi/3\} \times \{\sin x \times \cos 2\pi/3 + \cos x \times \sin 2\pi/3\}$$

$$= 4 \times \sin x \times \{(\sin x)/2 + (\sqrt{3} \times \cos x)/2\} \times \{-(\sin x)/2 + (\sqrt{3} \times \cos x)/2\}$$

$$= 4 \times \sin x \times \{-(\sin^2 x)/4 + (3 \times \cos^2 x)/4\}$$

$$= \sin x \times \{-\sin^2 x + 3 \times \cos^2 x\}$$

$$= \sin x \times \{-\sin^2 x + 3 \times (1 - \sin^2 x)\}$$

$$= \sin x \times \{-\sin^2 x + 3 - 3 \times \sin^2 x\}$$

$$= \sin x \times \{3 - 4 \times \sin^2 x\}$$

$$= 3 \times \sin x - 4\sin^3 x$$

$$= \sin 3x$$

$$\text{So, } 4 \times \sin x \times \sin(x + \pi/3) \times \sin(x + 2\pi/3) = \sin 3x$$

Question 4.

If $\tan x = (\cos 9 + \sin 9)/(\cos 9 - \sin 9)$, then $x =$

- (a) 45
- (b) 54
- (c) 36
- (d) None of these

Answer: (b) 54

Given, $\tan x = (\cos 9 + \sin 9)/(\cos 9 - \sin 9)$

$$\Rightarrow \tan x = \{\cos 9(1 + \sin 9/\cos 9)\}/\{\cos 9(1 - \sin 9/\cos 9)\}$$

$$\Rightarrow \tan x = (1 + \tan 9)/(1 - \tan 9)$$

$$\Rightarrow \tan x = (\tan 45 + \tan 9)/(1 - \tan 45 \times \tan 9) \text{ \{since } \tan 45 = 1\}}$$

$$\Rightarrow \tan x = \tan(45 + 9) \text{ \{Apply } \tan(A + B) \text{ formula}\}}$$

$$\Rightarrow \tan x = \tan(54)$$

$$\Rightarrow x = 54$$

Question 5.

In a triangle ABC, $\sin A - \cos B = \cos C$, then angle B is

- (a) $\pi/2$
- (b) $\pi/3$
- (c) $\pi/4$
- (d) $\pi/6$

Answer: (a) $\pi/2$

Given, $\sin A - \cos B = \sin C$

$$\Rightarrow \sin A = \cos B + \sin C$$

$$\Rightarrow 2 \times \sin(A/2) \times \cos(A/2) = 2 \times \cos\{(B+C)/2\} \times \cos\{(B-C)/2\}$$

$$\Rightarrow 2 \times \sin(A/2) \times \cos(A/2) = 2 \times \cos\{\pi/2 - A/2\} \times \cos\{(B-C)/2\}$$

$$\Rightarrow 2 \times \sin(A/2) \times \cos(A/2) = 2 \times \sin(A/2) \times \cos\{(B-C)/2\}$$

$$\Rightarrow \cos(A/2) = \cos\{(B-C)/2\}$$

$$\Rightarrow A/2 = (B-C)/2$$

$$\Rightarrow A = B - C$$

$$\Rightarrow B = A + C$$

$$\Rightarrow B = \pi - B \text{ \{Since } A + B + C = \pi\}}$$

$$\Rightarrow 2B = \pi$$

$$\Rightarrow B = \pi/2$$

Question 6.

The value of $\cos 420^\circ$ is

- (a) 0
- (b) 1
- (c) $1/2$
- (d) $\sqrt{3}/2$

Answer: (c) $1/2$

$$\cos 420^\circ = \cos(360^\circ + 60^\circ) = \cos 60^\circ = 1/2$$

Question 7.

If in a triangle ABC, $\tan A + \tan B + \tan C = 6$ then the value of $\cot A \times \cot B \times \cot C$ is

- (a) $1/2$
- (b) $1/3$
- (c) $1/4$
- (d) $1/6$

Answer: (d) $1/6$

$$\text{Given } \tan A + \tan B + \tan C = 6$$

Now $\tan(A + B + C) = \{(\tan A + \tan B + \tan C) - \tan A \times \tan B \times \tan C\} / \{1 - (\tan A \times \tan B + \tan B \times \tan C + \tan A \times \tan C)\}$

We know that,

$$A + B + C = \pi$$

$$\Rightarrow \tan(A + B + C) = \tan \pi$$

$$\Rightarrow \tan(A + B + C) = 0$$

Now

$$0 = \{(\tan A + \tan B + \tan C) - \tan A \times \tan B \times \tan C\} / \{1 - (\tan A \times \tan B + \tan B \times \tan C + \tan A \times \tan C)\}$$

$$\Rightarrow \tan A + \tan B + \tan C - \tan A \times \tan B \times \tan C = 0$$

$$\Rightarrow \tan A + \tan B + \tan C = \tan A \times \tan B \times \tan C$$

$$\Rightarrow \tan A \times \tan B \times \tan C = 6$$

$$\Rightarrow (1/\cot A) \times (1/\cot B) \times (1/\cot C) = 6$$

$$\Rightarrow 1/(\cot A \times \cot B \times \cot C) = 6$$

$$\Rightarrow \cot A \times \cot B \times \cot C = 1/6$$

Question 8.

If $a \times \cos x + b \times \sin x = c$, then the value of $(a \times \sin x - b \times \cos x)^2$ is

(a) $a^2 + b^2 + c^2$

(b) $a^2 - b^2 - c^2$

(c) $a^2 - b^2 + c^2$

(d) $a^2 + b^2 - c^2$

Answer: (d) $a^2 + b^2 - c^2$

We have

$$(a \times \cos x + b \times \sin x)^2 + (a \times \sin x - b \times \cos x)^2 = a^2 + b^2$$

$$\Rightarrow c^2 + (a \times \sin x - b \times \cos x)^2 = a^2 + b^2$$

$$\Rightarrow (a \times \sin x - b \times \cos x)^2 = a^2 + b^2 - c^2$$

Question 9.

When the length of the shadow of a pole is equal to the height of the pole, then the elevation of source of light is

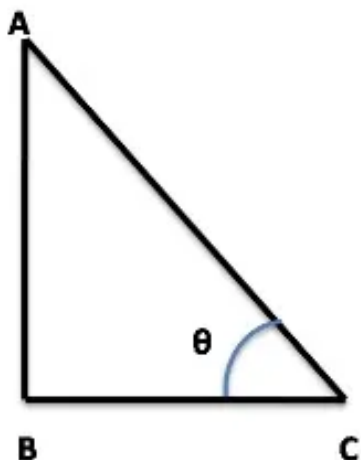
(a) 30°

(b) 60°

(c) 75°

(d) 45°

Answer: (d) 45°



Let AB is the length of the pole and BC is the shadow of the pole.

Given $AB = BC$

Now from triangle ABC,

$$\tan \theta = AB/BC$$

$$\Rightarrow \tan \theta = 1$$

$$\Rightarrow \theta = 45^\circ$$

So, the elevation of source of light is 45°

Question 10.

In any triangle ABC, if $\cos A/a = \cos B/b = \cos C/c$ and the side $a = 2$, then the area of the triangle is

- (a) $\sqrt{3}$
- (b) $\sqrt{3}/4$
- (c) $\sqrt{3}/2$
- (d) $1/\sqrt{3}$

Answer: (a) $\sqrt{3}$

Given $\cos A/a = \cos B/b = \cos C/c$

$$= \cos A/k \times \sin A = \cos B/k \times \sin B = \cos C/k \times \sin C \quad \{\text{since } \sin A/a = \sin B/b = \sin C/c = k\}$$

$$= \cot A = \cot B = \cot C$$

$$\Rightarrow A = B = C = 60^\circ$$

So, triangle ABC is equilateral.

$$\text{Now area of the triangle} = (\sqrt{3}/4) \times a^2 = (\sqrt{3}/4) \times 2^2 = (\sqrt{3}/4) \times 4 = \sqrt{3}$$

Question 11.

The least values of $\cos^2 \theta + \sec^2 \theta$ is

- (a) 0
- (b) 1

- (c) 2
(d) more than 2

Answer: (c) 2

If $a \times \cos^2 \theta + b \times \sec^2 \theta$ is given,

then the least value $= 2\sqrt{ab}$

Now given, $\cos^2 \theta + \sec^2 \theta$

Here, $a = 1$, $b = 1$

Now, least value $= 2\sqrt{(1 \times 1)} = 2 \times 1 = 2$

Question 12.

The equation $(\cos p - 1)x^2 + \cos p \times x + \sin p = 0$, where x is a variable, has real roots. Then the interval of p may be any one of the following:

- (a) $(0, \pi)$
(b) $(-\pi/2, \pi/2)$
(c) $(0, \pi)$
(d) $(-\pi, 0)$

Answer: (a) $(0, \pi)$

The equation $(\cos p - 1)x^2 + \cos p \times x + \sin p = 0$, where x is a variable, has real roots.

$x^2 + \cos p \times x + \sin p = 0$, where x is a variable, has real roots.

Now, for real roots,

Discriminant ≥ 0

$$\Rightarrow \cos^2 p - 4(\cos p - 1)\sin p \geq 0$$

$$\Rightarrow (\cos p - 2\sin p)^2 - 4\sin^2 p + 4\sin p \geq 0$$

$$\Rightarrow (\cos p - 2\sin p)^2 + 4\sin p(1 - \sin p) \geq 0 \dots\dots\dots 1$$

Now, $1 - \sin p \geq 0$

$\Rightarrow$ For all real p such that $0 < p < \pi$

So that $4\sin p(1 - \sin p) \geq 0$

So, $p \in (0, \pi)$

Question 13.

The value of $(\sec 8A - 1)/(\sec 4A - 1)$ is

- (a) 0
(b) 1
(c) $\tan 8A/\tan 2A$
(d) $\tan 2A/\tan 8A$

Answer: (c) $\tan 8A/\tan 2A$

Given, $(\sec 8A - 1)/(\sec 4A - 1)$

$$= (1/\cos 8A - 1)/(1/\cos 4A - 1)$$

$$\begin{aligned}
&= \{(1 - \cos 8A)/\cos 8A\} / \{(1 - \cos 4A)/\cos 4A\} \\
&= \{(1 - \cos 8A) \times \cos 4A\} / \{(1 - \cos 4A) \times \cos 8A\} \\
&= \{2\sin^2 4A \times \cos 4A\} / \{2\sin^2 2A \times \cos 8A\} \quad \{\text{since } \cos 2A = 1 - 2\sin^2 A\} \\
&= \{2\sin 4A \times \sin 4A \times \cos 4A\} / \{2\sin 2A \times \sin 2A \times \cos 8A\} \\
&= \{\sin 8A \times \sin 4A\} / \{2\sin 2A \times \sin 2A \times \cos 8A\} \quad \{\text{since } \sin 2A = 2 \times \sin A \times \cos A\} \\
&= \{\sin 8A \times 2\sin 2A \times \cos 2A\} / \{2\sin 2A \times \sin 2A \times \cos 8A\} \\
&= \{\sin 8A \times \cos 2A\} / \{\sin 2A \times \cos 8A\} \\
&= (\sin 8A / \cos 8A) / (\sin 2A / \cos 2A) \\
&= \tan 8A / \tan 2A \\
\text{So, } (\sec 8A - 1) / (\sec 4A - 1) &= \tan 8A / \tan 2A
\end{aligned}$$

Question 14.

The value of $(\sin 7x + \sin 5x) / (\cos 7x + \cos 5x) + (\sin 9x + \sin 3x) / (\cos 9x + \cos 3x)$ is

- (a) $\tan 6x$
- (b) $2 \tan 6x$
- (c) $3 \tan 6x$
- (d) $4 \tan 6x$

Answer: (b) $2 \tan 6x$

$$\begin{aligned}
&\text{Given, } (\sin 7x + \sin 5x) / (\cos 7x + \cos 5x) + (\sin 9x + \sin 3x) / (\cos 9x + \cos 3x) \\
&\Rightarrow [\{2 \times \sin(7x + 5x)/2 \times \cos(7x - 5x)/2\} / \{2 \times \cos(7x + 5x)/2 \times \cos(7x - 5x)/2\}] + \\
&[\{2 \times \sin(9x + 3x)/2 \times \cos(9x - 3x)/2\} / \{2 \times \cos(9x + 3x)/2 \times \cos(9x - 3x)/2\}] \\
&\Rightarrow [\{2 \times \sin 6x \times \cos x\} / \{2 \times \cos 6x \times \cos x\}] + [\{2 \times \sin 6x \times \cos x\} / \{2 \times \cos 6x \times \cos x\}] \\
&\Rightarrow (\sin 6x / \cos 6x) + (\sin 6x / \cos 6x) \\
&\Rightarrow \tan 6x + \tan 6x \\
&\Rightarrow 2 \tan 6x
\end{aligned}$$

Question 15.

If $x > 0$ then the value of $f(x) = -3 \times \cos \sqrt{3 + x + x^2}$ lie in the interval

- (a) $[-1, 1]$
- (b) $[-2, 2]$
- (c) $[-3, 3]$
- (d) None of these

Answer: (c) $[-3, 3]$

Given $x > 0$ then $3 + x + x^2 > 0$

Now, $-1 \leq \cos \sqrt{3 + x + x^2} \leq 1$ {Since $-1 \leq \cos x \leq 1$ }

$$\Rightarrow 3 \geq -3 \times \cos \sqrt{3 + x + x^2} \geq -3 \quad \{\text{Multiply by } -3\}$$

$$\Rightarrow -3 \leq f(x) \leq 3$$

$$\Rightarrow f(x) \in [-3, 3]$$

Question 16.

The value of $\cos 4A - \cos 4B$ is

- (a) $(\cos A - \cos B) \times (\cos A + \cos B) \times (\cos A - \sin B) \times (\cos A + \sin B)$
- (b) $2(\cos A - \cos B) \times (\cos A + \cos B) \times (\cos A - \sin B) \times (\cos A + \sin B)$
- (c) $4(\cos A - \cos B) \times (\cos A + \cos B) \times (\cos A - \sin B) \times (\cos A + \sin B)$
- (d) $8(\cos A - \cos B) \times (\cos A + \cos B) \times (\cos A - \sin B) \times (\cos A + \sin B)$

Answer: (d) $8(\cos A - \cos B) \times (\cos A + \cos B) \times (\cos A - \sin B) \times (\cos A + \sin B)$

Given, $\cos 4A - \cos 4B$

$$\begin{aligned} &= 2\cos^2 2A - 1 - (2\cos^2 2B - 1) \quad \{\text{since } 2\cos^2 x - 1 = \cos 2x\} \\ &= 2\cos^2 2A - 1 - 2\cos^2 2B + 1 \\ &= 2\cos^2 2A - 2\cos^2 2B \\ &= 2(\cos^2 2A - \cos^2 2B) \\ &= 2(\cos 2A - \cos 2B) \times (\cos 2A + \cos 2B) \\ &= 2\{2\cos^2 A - 1 - (2\cos^2 B - 1)\} \times \{2\cos^2 A - 1 + 1 - 2\sin^2 B\} \quad \{\text{since } 1 - 2\sin^2 x = \cos 2x\} \\ &= 2\{2\cos^2 A - 1 - 2\cos^2 B + 1\} \times \{2\cos^2 A - 1 + 1 - 2\sin^2 B\} \\ &= 2\{2\cos^2 A - 2\cos^2 B\} \times \{2\cos^2 A - 2\sin^2 B\} \\ &= 2 \times 2 \times 2\{\cos^2 A - \cos^2 B\} \times \{\cos^2 A - \sin^2 B\} \\ &= 8(\cos A - \cos B) \times (\cos A + \cos B) \times (\cos A - \sin B) \times (\cos A + \sin B) \\ \text{So, } \cos 4A - \cos 4B &= 8(\cos A - \cos B) \times (\cos A + \cos B) \times (\cos A - \sin B) \times (\cos A + \sin B) \end{aligned}$$

Question 17.

The value of $\cos 420^\circ$ is

- (a) 0
- (b) 1
- (c) $1/2$
- (d) $\sqrt{3}/2$

Answer: (c) $1/2$

$$\cos 420^\circ = \cos(360^\circ + 60^\circ) = \cos 60^\circ = 1/2$$

Question 18.

In a $\triangle ABC$, $(b + c) \cos A + (c + a) \cos B + (a + b) \cos C$ is equal to

- (a) $a + b + c$
- (b) 0
- (c) none of these
- (d) Rr

Answer: (a) $a + b + c$

Given $(b + c) \cos A + (c + a) \cos B + (a + b) \cos C$

$$= b \times \cos A + c \times \cos A + c \times \cos B + a \times \cos B + a \times \cos C + b \times \cos C$$

$$= (b \times \cos C + c \times \cos B) + (c \times \cos A + a \times \cos C) + (b \times \cos A + a \times \cos B)$$

$$= a + b + c \text{ \{since } b \times \cos C + c \times \cos B = a, c \times \cos A + a \times \cos C = b, b \times \cos A + a \times \cos B = c\}}$$

Question 19.

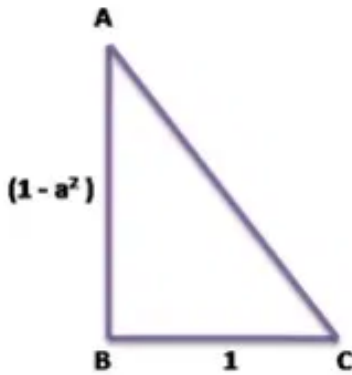
$\tan^2 \theta = 1 - a^2$ then the value of $\sec \theta + \tan^3 \theta \times \operatorname{cosec} \theta$ is

- (a) $(2 - a^2)$
- (b) $(2 - a^2)^{1/2}$
- (c) $(2 - a^2)^{3/2}$
- (d) None of these

Answer: (c) $(2 - a^2)^{3/2}$

Given, $\tan^2 \theta = 1 - a^2$

$$\Rightarrow \tan \theta = \sqrt{1 - a^2}$$



From the figure and apply Pythagoras theorem,

$$AC^2 = AB^2 + BC^2$$

$$\Rightarrow AC^2 = \{\sqrt{1 - a^2}\}^2 + 1$$

$$\Rightarrow AC^2 = 1 - a^2 + 1$$

$$\Rightarrow AC^2 = 2 - a^2$$

$$\Rightarrow AC = \sqrt{2 - a^2}$$

$$\text{Now, } \sec \theta = \frac{AC}{BC} = \frac{\sqrt{2 - a^2}}{1} = \sqrt{2 - a^2}$$

$$\operatorname{cosec} \theta = \frac{AC}{AB} = \frac{\sqrt{2 - a^2}}{\sqrt{1 - a^2}}$$

$$\text{and } \tan \theta = \sqrt{1 - a^2}$$

$$\text{Given, } \sec \theta + \tan^3 \theta \times \operatorname{cosec} \theta$$

$$= \sqrt{2 - a^2} + \{(1 - a^2)^{3/2} \times \sqrt{2 - a^2} / \sqrt{1 - a^2}\}$$

$$= \sqrt{2 - a^2} + \{(1 - a^2) \times (1 - a^2) \times \sqrt{2 - a^2} / \sqrt{1 - a^2}\}$$

$$= \sqrt{2 - a^2} + (1 - a^2) \times \sqrt{2 - a^2}$$

$$= \sqrt{2 - a^2} \times (1 + 1 - a^2)$$

$$= \sqrt{2 - a^2} \times (2 - a^2)$$

$$= (2 - a^2)^{3/2}$$

$$\text{So, } \sec \theta + \tan^3 \theta \times \operatorname{cosec} \theta = (2 - a^2)^{3/2}$$

Question 20.

The value of $\cos(\pi/7) \times \cos(2\pi/7) \times \cos(4\pi/7)$ is

- (a) $-1/2$
- (b) $-1/4$
- (c) $-1/6$
- (d) $-1/8$

Answer: (d) $-1/8$

We know that $\cos A \times \cos 2A \times \cos 2^2 A \times \dots \times \cos 2^{n-1} A = \sin(2^n A) / \{2^n \times \sin A\}$
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Given, $\cos(\pi/7) \times \cos(2\pi/7) \times \cos(4\pi/7)$

$$= \cos(\pi/7) \times \cos(2\pi/7) \times \cos(2^2 \pi/7)$$

$$= [\sin(2^3 \times \pi/7)] / \{2^3 \times \sin(\pi/7)\} \dots \dots \dots \text{from equation 1}$$

$$= [\sin(8\pi/7)] / \{8 \times \sin(\pi/7)\}$$

$$= [\sin(\pi + \pi/7)] / \{8 \times \sin(\pi/7)\}$$

$$= -\sin(\pi/7) / \{8 \times \sin(\pi/7)\}$$

$$= -1/8$$

$$\text{So, } \cos(\pi/7) \times \cos(2\pi/7) \times \cos(4\pi/7) = -1/8$$
